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AERATION MASS-TRANSFER RELATED TO REYNOLDS NUMBER

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A study of aeration rate data for streams and laboratory channels shows that the aeration rate can be correlated with Reynolds number and Schmidt number by a suitable dimensionless mass-transfer parameter which varies only slowly over a wide range of Reynolds number. This parameter, analogue to similar correlations in the heat- and mass-transfer fields, has two different near-constant levels, one in the low Reynolds number regime corresponding to molecular diffusivity, and another in the high Reynolds number regime corresponding to eddy diffusivity. The Schmidt number dependence has been found by comparison of gas into water data with vapour into gas data.

Introduction

In an instrument study, conducted for the Public Health Service[1] on aeration rate measurement, sufficient preliminary analysis suggested that the aeration rate could be correlated with Reynolds number and Schmidt number. This correlation is presented here in some detail.

The mass transfer of one fluid into another is commonly described by

$$M_{12} = KA'[c_1 - c_2] \quad \text{....................(1)}$$

Where M_{12} = mass transfer of component 1 into 2, across an interface

$c_{1,2}$ = 'concentrations' of components 1 and 2, appropriate to the process

K = a mass-transfer coefficient across the interface that fits the concentrations

A' = surface area at the interface

Such a conductance model is most suited to problems involving a hydrodynamic boundary layer, as in free or forced convection.

For example, in mass-transfer between similar phases, such as gas diffusing into gas, Jakob[2] uses a temperature-weighted concentration difference as the driving potential. This preserves the purity of a partial-pressure drive governing mass-transfer in the case of near ideal gases.

In the case of present interest, gas transferring into a flowing stream of water, the most suitable conductance description is by means of the Adeney entrance and exit coefficients.

$$M = A'[\mathrm{a}p - \mathrm{b}\rho] \quad \ldots\ldots\ldots\ldots(2)$$

Where M/A' = mass-transfer per unit area of gas into liquid
p = partial pressure of gas in the gas phase transferring mass into the liquid stream
ρ = concentration (i.e., gas density) of gas in the liquid phase
a = entrance coefficient (suiting the gas phase)
b = exit coefficient (suiting the liquid phase)

This has taken formal account of the differences in 'concentration potential' drives in the two phases.

It is common, at an equilibrium in which the liquid is incapable of taking up any further gas, to describe the final liquid concentration ρ_s as the 'saturation' value, i.e.

$$\mathrm{a}p = \mathrm{b}\rho_s \quad \ldots\ldots\ldots\ldots(3)$$

The applicability of this result in the more general mass-transfer case generally slurs over the question of the constancy of a and b. If a and b are constant, then

$$M/A' = \mathrm{b}(\rho_s - \rho) \quad \ldots\ldots\ldots\ldots(4)$$

In this form, subject to the stated conditions, the coefficient b may be identified as the mass-transfer coefficient, K_L.

$$M/A' = K_L(\rho_s - \rho) \quad \ldots\ldots\ldots\ldots(5)$$

(That these are real difficulties in the latter definition may be noted in Dorsey[3] who calls attention to the fact that neither a nor b appears to be constant in transient saturation of solutions. These remarks are directed toward anticipating difficulties to be found in surface active impurities at the liquid–gas interface. [See references 4 and 5]. However, for many common cases of mass-transfer of gas liquid, essentially constant Adeney coefficients are found.)

If the aeration mass-transfer takes place into a flowing stream of depth H (i.e., transfer in the case of a forced convective field), one might expect a correlation of the form.

$$K_L H/\nu = f_1(\mathrm{Re}, \mathrm{S}) \quad \ldots\ldots\ldots\ldots(6)$$

where $K_L = K_2 H = H k_2/0{\cdot}434$
K_2, k_2 = aeration rate
K_L = mass-transfer coefficient
H = depth of stream
ν = kinematic viscosity
Re = Reynolds number
S = Schmidt number
f_1 = as yet unspecified function of Re and S

Since one might expect that in a diffusive process at right angles to a flow process the diffusion would be dependent on the square root of velocity (for example, a dye introduced into a stream travels downstream with stream velocity, but diffuses laterally according to the square root of the velocity, both in a low-velocity laminar-flow field and in a high-velocity eddy-diffusive field), it would appear most suitable to use a new parameter β

$$\beta = K_L H/\nu \mathrm{Re}^{1/2} = f(\mathrm{Re}, \mathrm{S}) \quad \ldots\ldots\ldots\ldots(7)$$

If the correlation idea were successful, then this parameter would have utility similar to the friction factor in fluid streams.

If an elementary correlation holds, this parameter, β, should be constant, or very nearly so, over a wide range of Reynolds number. (However, the levels might be expected to be different in molecular diffusive and eddy diffusive flows.)

This form of β was postulated for comparison of river data. However, much of the experimental work in the literature is for laboratory channels which are not similar to rivers, in that river channels are very wide compared with their depth. To correlate data from such diverse channels, the form of β must be modified. The nature of a suitable modification may be determined by noting how the Reynolds number is correlated for different channel shapes.

The Reynolds number, for a stream, may be defined as

$$\mathrm{Re} = HV/\nu \quad \ldots\ldots\ldots\ldots(8)$$

H = depth of the stream
V = mean velocity of the stream
ν = kinematic viscosity of the stream

To take into account widely different section geometries, it is more general to correlate on the basis of the hydraulic radius, R, i.e.

$$\mathrm{Re} = RV/\nu \quad \ldots\ldots\ldots\ldots(9)$$

where $V = Q/A$ and $R = A/p$
Q = volume flow of the stream
p = wetted perimeter of the cross-section in contact with walls
A = cross-sectional area of stream

Thus,

$$\mathrm{Re} = RV/\nu = Q/p\nu \quad \ldots\ldots\ldots\ldots(10)$$

furnishes equivalent definitions in terms of either flow or mean velocity.

Similarly, the most appropriate generalisation would seem to be

$$\beta = K_L R/\nu \mathrm{Re}^{1/2} \quad \ldots\ldots\ldots\ldots(11)$$

leaving

$$K_L = K_2 H \quad \ldots\ldots\ldots\ldots(12)$$

regardless of channel shape.

It was found that this correlating choice was consistent with data of Streeter on channels of different width to depth ratios.

It is common in fluid mechanical correlation to find separable functions, so that one might expect

$$\beta = K_L R/\nu \mathrm{Re}^{1/2} = f(\mathrm{Re}) \cdot g(\mathrm{S}) \quad \ldots\ldots\ldots\ldots(13)$$

in which f and g are distinct functions. It is then further expected that a Schmidt number dependence in a power form may exist

$$\beta = f(\mathrm{Re})/\mathrm{S}^m \quad \ldots\ldots\ldots\ldots(14)$$

where m is likely to be a slowly varying exponent with possible range $\frac{1}{3}$ to $\frac{2}{3}$. The functional dependence of Schmidt number may alternatively be noted as a temperature function,

$$\beta = f(\mathrm{Re})/\mathrm{S}T^m \quad \ldots\ldots\ldots\ldots(14)$$

where T = temperature

The reason for choosing normalisation by viscosity ν, a momentum diffusive process, rather than mechanical diffusivity δ, is that the dependence on Schmidt number is unknown, and in fact unproven. By using ν and probing the Schmidt number relation, the validity of a diffusive model can be established.

Analysis of the data

The following experimental data (Tables I–VI) were analysed for correlation by the dimensionless aeration parameter β.

TABLE I

Re-aeration rates from river data (O'Connor & Dobbins[6])

River	H, ft	V, ft/sec	T, °C	k_2, day^{-1}	ν, ft^2/h	β	Re
Elk	0·9	0·97	12	4·8	0·0480	0·030	66,000
Clarion	1·9	0·55	13	2·6	0·0465	0·068	81,000
Tennessee	4·0	0·73	23	1·12	0·0364	0·088	290,000
Illinois	9·2	1·37	27	0·27	0·0330	0·057	1,380,000
Illinois	9·0	1·57	27	0·32	0·0330	0·061	1,540,000
Illinois	8·9	1·63	27	0·35	0·0330	0·064	1,580,000
San Diego Bay	12·0	0·32	20	0·048	0·0390	0·029	360,000
San Diego Bay	32·0	0·50	20	0·018	0·0390	0·037	1,480,000
San Diego Bay	37·0	0·88	20	0·026	0·0390	0·050	3,000,000
Ohio	15·0	4·20	16·3	0·26	0·0431	0·057	5,300,000
Ohio	14·3	3·68	16·8	0·77	0·0420	0·169	4,500,000
Ohio	18·7	3·72	16·7	0·21	0·0420	0·068	6,000,000
Ohio	24·2	3·90	17·3	0·32	0·0420	0·150	8,100,000

Approximate mean Schmidt number: 500

Note: The Ohio River data are only a sampling of the points

TABLE II

Re-aeration rates from laboratory channels (Streeter, Wright & Kehr[7])

H, in.	Q, gal/min.	w, in.	k_2, day^{-1}	T, °C	ν, cm^2/sec	P, in.	A, in.^{2}	β	Re
4	0·75	5	—	10	0·0131	13	20	—	110
4	0·75	5	0·42	10	0·0131	13	20	0·0033	110
4	0·75	2	0·40	10	0·0131	10	8	0·00142	140
4	0·75	2	0·38	9·7	0·0132	10	8	0·00134	140
4	0·75	2	0·38	9·5	0·0133	10	8	0·00133	140
4	0·75	2	0·37	9·5	0·0133	10	8	0·00130	140
4	0·75	2	0·40	9·5	0·0133	10	8	0·00142	140
4	0·75	2	0·40	9·5	0·0133	10	8	0·00141	140
Approximate Schmidt number: 850									
Temperatures: 0—10° (Average temp. used)									
2	0·5	2	0·53		0·0152	6	4	0·00068	140
3	0·5	2	0·37		0·0152	8	6	0·00093	100
4	0·5	2	0·26		0·0152	10	8	0·00105	80
2	0·75	2	0·71		0·0152	6	4	0·00075	200
3	0·75	2	0·46		0·0152	8	6	0·00095	150
4	0·75	2	0·20		0·0152	10	8	0·00066	120
2	1·5	2	0·91		0·0152	6	4	0·00068	410
3	1·5	2	0·54		0·0152	8	6	0·00079	310
4	1·5	2	0·42		0·0152	10	8	0·00109	200
Approximate Schmidt number: 1100									
Temperatures: 10—20° (Average temp. used)									
2	0·5	2	0·88		0·0114	6	4	0·0013	180
3	0·5	2	0·56		0·0114	8	6	0·0016	140
4	0·5	2	0·34		0·0114	10	8	0·0016	110
2	0·75	2	1·06		0·0114	6	4	0·0013	270
3	0·75	2	0·69		0·0114	8	6	0·0016	200
4	0·75	2	0·53		0·0114	10	8	0·0020	160
2	1·5	2	1·52		0·0114	6	4	0·0013	540
3	1·5	2	0·93		0·0114	8	6	0·0016	410
4	1·5	2	0·69		0·0114	10	8	0·0019	330
Approximate Schmidt number: 600									
Temperatures: 20—30° (Average temp. used)									
2	0·5	2	1·15		0·0089	6	4	0·0019	230
2	0·75	2	1·65		0·0089	6	4	0·0023	350
3	0·75	2	1·01		0·0089	8	6	0·0027	260
4	0·75	2	0·82		0·0089	10	8	0·0035	210
2	1·5	2	2·61		0·0089	6	4	0·0024	700
3	1·5	2	1·32		0·0089	8	6	0·0025	520
4	1·5	2	1·16		0·0089	10	8	0·0035	420

Approximate Schmidt number: 350

TABLE III

H, in.	w, in.	V, ft/min.	k_2, day^{-1}	T, °C	P, in.	A, in.^{2}	ν, cm^2/sec	β	Re
4	3	2·6	1·18	12·0	11	12	0·0124	0·0042	300
4	3	3·3	1·44	14·3	11	12	0·0116	0·0047	400
4	3	6·9	2·23	14·7	11	12	0·0115	0·0050	840
4	3	14·0	5·2	15·7	11	12	0·0112	0·0083	1660
4	3	14·0	5·9	18·7	11	12	0·0104	0·0098	1900
4	3	23·8	11·4	14·0	11	12	0·0117	0·0137	2900
4	3	40·0	36·2	12·8	11	12	0·0121	0·033	4700
12	6	6·0	0·66	21·0	30	72	0·0098	0·0076	1900
12	6	9·3	1·36	23	30	72	0·0094	0·0129	3100
12	6	18·0	3·43	26	30	72	0·0088	0·025	6300
12	6	24·5	4·37	20·1	30	72	0·0101	0·025	7500
12	6	34·0	6·3	20	30	72	0·0101	0·029	11400
12	6	47·0	9·35	25	30	72	0·0090	0·040	16200

Approximate Schmidt number: 600 for data around 15°
500 for data around 20°

TABLE IV

Re-aeration rate from laboratory channels (Kehr[4])
Width 6 in.; depth 12 in. (a); tap water results (b)

V, ft/min.	r_0, g/m^{2}day^{-1}	T, °C	ν, cm^2/sec	K_L, cm/h	β	Re
22·2	25·2	20	0·0101	11·4	0·023	6800
22·2	25·2	20	0·0101	11·4	0·023	6800
22·0	25·0	20	0·0101	11·3	0·023	6800
21·2	24·0	20	0·0101	10·9	0·023	6500
21·6	24·4	20	0·0101	11·1	0·023	6700
21·6	24·4	20	0·0101	11·1	0·023	6700
21·6	25·1	20	0·0101	11·4	0·023	6700
21·6	24·4	20	0·0101	11·1	0·023	6700
5·0	3·6	25	0·00895	1·79	0·0082	1700
4·7	3·2	25	0·00895	1·59	0·0075	1600
5·3	3·8	25	0·00895	1·89	0·0084	1800
5·2	3·9	25	0·00895	1·94	0·0087	1800
10·7	10·4	25	0·00895	5·2	0·016	3700
9·9	9·8	25	0·00895	4·9	0·016	3400
11·3	11·4	25	0·00895	5·7	0·017	3900
9·8	9·7	25	0·00895	4·8	0·016	3400
21·4	25·8	25	0·00895	12·8	0·028	7400
20·9	25·0	25	0·00895	12·4	0·028	7200
20·5	24·8	25	0·00895	12·3	0·028	7100
23·1	28·5	25	0·00895	14·2	0·030	8000
29·3	37·8	25	0·00895	18·8	0·035	10,100
28·3	36·3	25	0·00895	18·0	0·034	9800

Approximate Schmidt number; 500

(a) The channel depth appears to have been 12 in. (This value was assumed from reference 7, p. 300, which states that tap water experiments were done with water depth of 12 in. in the larger recirculating channel, and reference 8, p. 1515, which states that the results of both Streeter[7] and Kehr[4] were taken in a channel operated at 12 in. depth.). Then $K_L = r_0/\rho_s$. ρ_s $(= D_0)$ = saturated concentration in g/cm^3 or p.p.m., taken from an early Part II[7a] of which is Part III[7].

(b) The data used were obtained with de-aerated tap water. The paper also gives re-aeration rate data for various practical contaminants such as sewage concentrations of 0—25%; sludge concentrations of 10—100 p.p.m. dry solids; and a variety of common waste water materials (urine, oils, soaps, fatty acids). These materials show reduced aeration rates that vary with the material and the velocity range. The re-aeration rates appear to be in the range 40—80% of the tap water rate.

TABLE V

Re-aeration rates from San Diego Bay (Nusbaum & Miller[8])

H, ft	V, ft/min.	A, ft^2	K_L (at 20°), cm/h	ν, ft^2/h	β	Re
37	53	144×10^6	2·20	0·0388	0·040	$3·0\times10^6$
32	30	$76·5\times10^6$	1·31	0·0388	0·029	$1·5\times10^6$
12	19	$102·5\times10^6$	1·34	0·0388	0·023	$3·5\times10^5$
7	15	$161·5\times10^6$	1·45	0·0388	0·021	$1·6\times10^5$

Approximate Schmidt number: 500

TABLE VI

Re-aeration rates from laboratory channels (Krenkel & Orlob[9])
(Data reduced to 20°)

H, ft	E^a, ft^2/min.^{3}	$k_2H^{0\cdot66}$ ft$^{0\cdot66}$/min.	k_2, min.$^{-1}$	Re	β
0·0901	1000	0·0020	0·0090	4000	0·0045
0·0901	3000	0·0030	0·0147	6100	0·0054
0·9001	35,000	0·0080	0·0392	14,800	0·0093
0·0901	300,000	0·020	0·098	32,000	0·0158

One Datum in Text

H, ft	V, ft/sec	k_2 min.$^{-1}$	Re	β
0·0901	1·34	0·0485	11,200	0·0132

Approximate Schmidt number: 500

a The authors' Fig. 13 presents 'energy expenditure' E in ft^2/min.^{3} vs. $k_2H^{0\cdot66}$, where k_2 represents a temperature-corrected aeration coefficient. The authors relate the energy to the mean velocity by

$$E = Vsg$$

V = mean stream velocity
E = energy dissipation per unit mass of fluid
s = slope of energy gradient
g = normal gravity

Since this involves the second unknown of channel slope, the Chezy formula

$$V = C\,(H.S)^{1/2}$$

was assumed. The Powell formula—for smooth channels—was assumed for the Chezy constant C

$$C = 42 \log_{10}4\mathrm{Re}/C$$

The computational procedure for the above table involved selecting energy values that covered the experimental range in Fig. 13; picking off the corresponding values of $k_2H^{0\cdot66}$; computing k_2 (assuming a constant height in all experiments); computing Re from

$$\mathrm{Re} = H/\nu\cdot BC^2H^{1/3}/9$$

and then computing β from Equation (7).
There is one direct datum given in the text. This is also shown. A moderate difference represents, essentially, a difference in choice of Chezy constant.

TABLE VII

Re-aeration values supplied by Dr. P. Krenkel (private communication)

H, ft	V, ft/min.	k_2, min.$^{-1}$	T, °C	ν, cm^2/sec	β	Re
0·0802	28·36	0·0245	21·1	0·0098	0·0096	3600
0·0823	44·34	0·0482	19·7	0·0101	0·0154	5600
0·0820	23·10	0·0272	22·1	0·0096	0·0123	3000
0·0824	17·70	0·0176	21·2	0·0098	0·0091	2300
0·0861	47·27	0·0464	21·6	0·0097	0·0158	6500
0·0852	53·80	0·0579	20·5	0·0099	0·0179	7200
0·0857	61·18	0·0682	19·5	0·0102	0·0197	8000
0·0844	38·04	0·0347	19·2	0·0103	0·0123	4800
0·0871	49·38	0·0559	22·5	0·0095	0·0191	7000
0·0894	27·66	0·0289	21·6	0·0097	0·0136	3900
0·0930	79·80	0·0738	20·8	0·0099	0·0215	1160
0·0901	72·30	0·0822	21·1	0·0098	0·0240	10,300
0·0878	33·18	0·0246	21·1	0·0098	0·0102	4600
0·0932	21·84	0·0191	21·5	0·0097	0·0107	3300
0·0876	61·44	0·0727	21·6	0·0097	0·0222	8600
0·0882	63·78	0·0810	21·8	0·00965	0·0246	9000
0·0964	51·54	0·0438	22·7	0·00945	0·0171	8100
0·0971	61·20	0·0576	22·1	0·0096	0·0207	9590
0·0974	69·00	0·0564	22·1	0·0096	0·0192	10,800
0·0974	36·30	0·0294	21·6	0·0097	0·0137	5600
0·0963	71·10	0·0565	21·6	0·0097	0·0184	11,000
0·1091	72·00	0·0467	20·0	0·0101	0·0179	12,100
0·1115	39·74	0·0200	19·6	0·0102	0·0106	6700
0·0993	53·88	0·0478	21·3	0·00975	0·0187	8500
0·1001	14·58	0·0105	22·3	0·00955	0·0081	2400
0·1048	47·58	0·0350	20·9	0·00985	0·0157	7900
0·1045	76·32	0·0736	21·4	0·00975	0·0262	12,600
0·1090	95·40	0·0829	21·6	0·0097	0·0281	16,600
0·1043	86·28	0·0780	21·2	0·0098	0·0260	14,300
0·1232	49·80	0·0319	22·1	0·0096	0·0182	9900
0·1182	58·02	0·0448	21·2	0·0098	0·0219	10,900
0·1161	65·10	0·0528	22·4	0·0095	0·0241	12,300
0·1189	66·48	0·0474	22·5	0·0095	0·0222	12,900
0·1360	95·40	0·0708	22·5	0·0095	0·0338	21,000
0·1416	32·34	0·0146	22·8	0·00945	0·0128	7500
0·1422	43·98	0·0241	22·6	0·0095	0·0181	10,200
0·1415	54·25	0·0307	22·7	0·00945	0·0207	12,600
0·1427	87·00	0·0662	19·9	0·0101	0·0346	19,000
0·1410	90·60	0·0581	20·5	0·00995	0·0294	20,000
0·1447	21·54	0·0098	22·6	0·0095	0·0108	5100
0·1463	70·80	0·0388	22·2	0·00955	0·0241	16,800
0·1451	80·40	0·0485	20·5	0·00995	0·0272	18,200
0·1478	61·80	0·0384	21·6	0·00965	·00257	14,600
0·1521	108·60	0·0717	23·0	0·0094	0·0382	27,200
0·1507	121·80	0·0712	22·3	0·00955	0·0352	29,700
0·1481	128·40	0·0799	22·4	0·0095	0·0374	31,100
0·1650	24·48	0·0097	22·9	0·0094	0·0123	6700
0·1651	63·00	0·0277	22·0	0·0096	0·0217	16,800
0·1650	81·00	0·0382	22·4	0·0095	0·0265	21,800
0·1662	93·60	0·0389	22·1	0·0096	0·0253	25,100
0·1651	49·68	0·0234	21·8	0·00965	0·0206	13,200
0·2014	26·46	0·0075	20·7	0·0099	0·0126	8400
0·1983	44·58	0·0121	22·4	0·0095	0·0149	14,400
0·1985	71·16	0·0282	21·5	0·0097	0·0272	22,600
0·1989	61·20	0·0208	21·6	0·0097	0·0217	19,500
0·1995	78·90	0·0273	21·9	0·0096	0·0254	25,400
0·1968	88·68	0·0312	21·9	0·0096	0·0268	28,100
0·1989	107·40	0·0365	21·7	0·0097	0·0287	34,200

The data in more detail in Table VII (not graphed) were graciously supplied by Dr. Peter Krenkel; the results in Tables VIII and IX were obtained by other authors.

The results from these tables have been plotted in Fig. 1 in log–log form as β vs. Re. The data suggest a sweeping S-shaped change from a near-constant level for β at low Reynolds number, a fairly broad transition from relatively low Reynolds number (perhaps of the order of 100) to a near-constant level for β at high Reynolds number (perhaps above 10^5). The low Reynolds number values are taken from the early work of Streeter *et al.*[7] working with artificial channels. The high Reynolds number data are taken from the work of various authors,[6,10,11] which are results from natural streams. The papers of Kehr[4] and Krenkel & Orlob,[9] which are from laboratory channels, contributed values for the transitional area.

The hypothesis of an S-shaped form for the β curve is based on the idea that the processes in all cases are 'diffusive', ranging from a molecular diffusivity at low stream velocities, to an eddy diffusivity at high stream velocities. In both cases, one might expect a lateral propagation of mass flux according to the square root of the streaming velocity; thus, making nearly a constant. However, the scale in the two regimes would be considerably different. Fig. 1 suggests a difference in level of the order of a factor of 100, which is not too unreasonable on an intuitive basis of experience with heat- and mass-transfer fields. Thus, Fig. 1 appears well founded, but it is evident that there is a considerable scatter in the data, perhaps as much as a half decade. Three possible reasons may be suggested for this scatter: difficult experimental problems; no semi-theoretical guide like Fig. 1 to refer to as a standard; or an intrinsic large variation of aeration coefficient with 'impurities' in the water. Previous experience with hydrodynamic correlations like Fig. 1 has generally demonstrated that the appearance of such a correlation helps to orient subsequent work until a more precise idea can finally emerge on the intrinsic variation due to other than experimental errors (i.e., to whatever might cause systematic sources of error).

The problem of the effect of contaminants on aeration rate is not quite so simple, as earlier comments have indicated. The most logical formulation of the aeration problem is in terms of the Adeney entrance and exit coefficients as in Equation (2) above.

It was pointed out from data presented in Dorsey[3] and the work of Pasveer,[12] that the entrance and exit coefficients are not necessarily constants. In general, the approximate value of a and b sought are ones that will be operationally most appropriate for oxygen sags in streams.

TABLE VIII

Re-aeration rates from river data (reduced to 20°) (Churchill, Elmore, & Buckingham[10])

H, ft	Q, ft^3/sec	w, ft	V, ft/sec	k_2, day^{-1}	T, °C	ν, cm^2/sec	β	Re	S
3·27	2300	229	3·07	2·272	13·9	0·0118	0·057	$8\cdot0\times10^{5}$	670
5·09	4550	242	3·69	1·440	14·2	0·0117	0·065	$1\cdot5\times10^{6}$	660
4·42	3190	344	2·10	0·981	11·4	0·0126	0·045	$6\cdot9\times10^{5}$	770
6·14	5890	358	2·68	0·496	11·5	0·0125	0·034	$1\cdot2\times10^{6}$	770
5·66	5910	376	2·78	0·743	11·9	0·0124	0·043	$1\cdot2\times10^{6}$	740
7·17	5930	313	2·64	1·129	12·0	0·0124	0·098	$1\cdot4\times10^{6}$	730
11·41	10,385	312	2·92	0·281	17·7	0·0107	0·049	$2\cdot9\times10^{6}$	540
2·12	3230	617	2·47	3·361	15·5	0·0113	0·051	$4\cdot4\times10^{5}$	600
2·93	6400	635	3·44	2·794	15·8	0·0112	0·058	$8\cdot3\times10^{5}$	600
4·54	14,085	667	4·65	1·568	19·8	0·0101	0·057	$1\cdot9\times10^{6}$	480
9·50	10,440	374	2·94	0·455	17·8	0·0106	0·062	$2\cdot4\times10^{6}$	530
6·29	6540	414	2·51	0·389	17·8	0·0106	0·031	$1\cdot4\times10^{6}$	540
7·52	10,500	443	3·15	0·270	18·4	0·0104	0·025	$2\cdot1\times10^{6}$	520
7·07	10,500	450	3·30	0·550	18·4	0·0104	0·045	$2\cdot1\times10^{6}$	520
5·44	5590	330	3·11	0·544	17·8	0·0106	0·031	$1\cdot5\times10^{6}$	540
8·06	11,903	346	4·28	0·604	20·4	0·0100	0·055	$3\cdot2\times10^{6}$	470
3·88	11,952	90	2·73	1·251	17·8	0·0106	0·046	$9\cdot2\times10^{5}$	540
9·28	12,010	537	2·41	0·273	25·6	0·0089	0·043	$2\cdot3\times10^{6}$	360
10·19	17,120	549	3·06	0·225	26·0	0·0088	0·036	$3\cdot3\times10^{6}$	360
3·29	4105	520	2·40	1·881	25·4	0·0089	0·062	$8\cdot3\times10^{5}$	360
4·74	8775	535	3·46	0·842	25·3	0·0089	0·040	$1\cdot7\times10^{6}$	370
5·72	12,455	542	4·02	0·883	26·3	0·0087	0·052	$2\cdot5\times10^{6}$	350
6·95	17,270	550	4·52	0·915	25·8	0·0088	0·069	$3\cdot3\times10^{6}$	360
4·29	4105	517	1·85	0·995	25·4	0·0089	0·056	$8\cdot3\times10^{5}$	360
6·01	8775	531	2·75	0·547	25·3	0·0089	0·042	$1\cdot7\times10^{6}$	370
7·16	12,455	539	3·23	0·881	26·0	0·0088	0·081	$2\cdot5\times10^{6}$	360
8·49	17,270	548	3·71	0·252	25·8	0·0088	0·028	$3\cdot3\times10^{6}$	360
3·42	3112	182	5·00	5·558	10·8	0·0128	0·11	$1\cdot2\times10^{6}$	800
3·02	1145	124	3·05	1·712	23·4	0·0093	0·043	$9\cdot2\times10^{5}$	400
2·83	1145	115	3·51	3·222	23·4	0·0093	0·069	$1\cdot0\times10^{6}$	400

Approximate mean Schmidt number: 500

Note: Tabulated Reynolds number from text. Thus H is consistently used instead of R in Re and β, but viscosity is based on observed temperature.

TABLE IX

Re-aeration rates from Ohio River data (Streeter & Phelps[11])
(Reduced to 20° values)

H, ft	V, ft/sec	k_2, day^{-1}	β	Re	H, ft	V, ft/sec	k_2, day^{-1}	β	Re
16·3	0·5	0·33	0·25	$7\cdot5\times10^{5}$	3·2	1	0·14	0·0065	$3\cdot0\times10^{5}$
16·3	1	0·50	0·27	$1\cdot5\times10^{6}$	5·7	2	0·28	0·022	$1\cdot1\times10^{6}$
16·3	2	0·74	0·28	$3\cdot0\times10^{6}$	9·2	3	0·31	0·040	$2\cdot5\times10^{6}$
16·3	3	0·92	0·28	$4\cdot5\times10^{6}$	19·2	4	0·15	0·051	$7\cdot1\times10^{6}$
19·0	4	0·80	0·27	$7\cdot0\times10^{6}$	4·0	2	0·24	0·011	$7\cdot4\times10^{5}$
4·4	0·5	0·20	0·021	$2\cdot0\times10^{5}$	8·4	3	0·28	0·032	$2\cdot3\times10^{6}$
6·6	1	0·27	0·037	$6\cdot1\times10^{5}$	22·2	4	0·13	0·055	$8\cdot2\times10^{6}$
10·0	2	0·35	0·064	$1\cdot8\times10^{6}$	10·3	0·5	0·17	0·065	$4\cdot7\times10^{5}$
12·8	3	0·40	0·086	$3\cdot5\times10^{6}$	11·1	1	0·49	0·15	$1\cdot0\times10^{6}$
15·8	4	0·42	0·11	$5\cdot8\times10^{6}$	13·0	2	1·16	0·31	$2\cdot4\times10^{6}$
3·9	0·5	0·17	0·015	$1\cdot8\times10^{5}$	19·9	3	0·99	0·41	$5\cdot5\times10^{6}$
5·0	1	0·25	0·023	$4\cdot6\times10^{5}$	31·1	4	0·66	0·47	$1\cdot1\times10^{7}$
7·2	2	0·31	0·034	$1\cdot3\times10^{6}$	1·8	1	0·07	0·0014	$1\cdot7\times10^{5}$
9·4	3	0·32	0·043	$2\cdot6\times10^{6}$	8·8	2	0·12	0·018	$1\cdot6\times10^{6}$
12·0	4	0·30	0·051	$4\cdot4\times10^{6}$	15·2	3	0·38	0·11	$4\cdot2\times10^{6}$
5·2	0·5	0·70	0·095	$2\cdot4\times10^{5}$	24·8	4	0·63	0·31	$9\cdot2\times10^{6}$
7·0	1	0·78	0·12	$6\cdot5\times10^{5}$	1·6	0·5	1·38	0·032	$7\cdot4\times10^{4}$
8·5	2	1·05	0·15	$1\cdot6\times10^{6}$	5·4	1	0·62	0·063	$5\cdot0\times10^{5}$
10·1	3	1·12	0·17	$2\cdot8\times10^{6}$	12·1	2	0·62	0·15	$2\cdot2\times10^{6}$
12·4	4	0·99	0·18	$4\cdot6\times10^{6}$	18·2	3	0·72	0·26	$5\cdot0\times10^{6}$
4·9	0·5	0·27	0·034	$2\cdot3\times10^{5}$	25·5	4	0·72	0·38	$9\cdot4\times10^{6}$
5·8	1	0·54	0·061	$5\cdot4\times10^{5}$	4·8	0·5	0·38	0·046	$3\cdot3\times10^{5}$
7·1	2	1·00	0·11	$1\cdot3\times10^{6}$	7·8	1	0·62	0·11	$7\cdot2\times10^{5}$
9·0	3	1·14	0·14	$2\cdot5\times10^{6}$	14·6	2	0·78	0·25	$2\cdot7\times10^{6}$
11·6	4	1·04	0·17	$4\cdot3\times10^{6}$	22·0	3	0·81	0·39	$6\cdot1\times10^{6}$
0·9	0·5	0·29	0·0028	$4\cdot1\times10^{4}$	33·4	4	0·64	0·50	$1\cdot2\times10^{7}$

Approximate mean Schmidt number: 500

The immediate identification of a as in Equation (3) above does not serve to establish the less-than-saturated entrance coefficient. Furthermore, from papers such as Kehr's[4] or Downing *et al.*[5], it is not clear that all of the quantities in the definition of entrance coefficient have been explored carefully as a function of what appears to be surface active contamination. What appears to be the case in both references 4 and 5 is a reduction to values of aeration coefficients to 40—80% of 'pure' tap water results. These effects cannot be readily observed systematically in Fig. 1. This would require separate study of aeration rate as a function of some surface-active property such as surface tension, as well as time.

At first glance an asymptotic value of β at low Reynolds number might be expected if molecular diffusivity were the major governing process. However, it is more likely that at low Reynolds number a 'diffusive' result should fan out to a family of free convective values for β which may or may

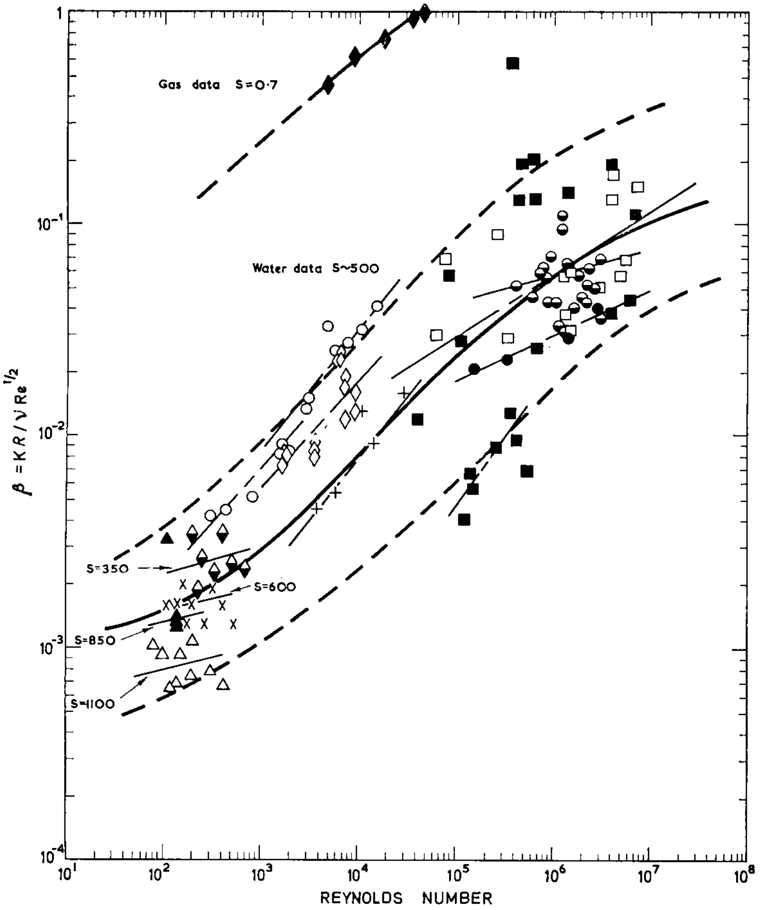

Fig. 1. *Correlation of cross-stream mass transfer with the stream forced convection*

β = dimensionless mass transfer coefficient
Re = Reynolds no. of the streams
S = Schmidt no. of the interdiffusing species
R = hydraulic radius.

Approx. Schmidt no. dependence estimated at Re = 10,000
$\beta_1/\beta_2 = (S_2/S_1)^m$ or $0{\cdot}60/0{\cdot}008 = (500/0{\cdot}7)^m$
Whence $m = 0{\cdot}66$.

□ O'Connor & Dobbins[6] S ~ 500
▲ Streeter *et al.*[7] S ~ 850
△ „ „ S ~ 1100
× „ „ S ~ 600
⬙ „ „ S ~ 350
○ „ „ S ~ 500 and 600
◇ Kehr[4] S ~ 500
● Nusbaum & Miller[8] S ~ 500
+ Krenkel & Orlob[9] S ~ 500
◒ Churchill *et al.*[10] S ~ 500
■ Streeter & Phelps[11] S ~ 500
◆ Sherwood & Woertz[13] S ~ 0·7

not be near-constant. It would perhaps be useful to have a theoretical guide for the low Reynolds number end.

The remaining task that must be done in the correlation is to determine the effect of Schmidt number, under the assumption that it acts to determine the aeration rate transfer coefficient as a product factor.

In Fig. 1, on aeration rate transfer into a water medium, the effect of Schmidt number would be shown, in whole or part, as a temperature effect. Accordingly, as far as possible, the data were segregated by temperatures.

However, both the scatter in the data and the limited range of Schmidt number in water (at most from a few hundred to a thousand) make the correlation difficult. It is only the early Streeter data[7] that seem to warrant any segregation of the data to temperature. The other data can be identified only as nominal 20°, Schmidt number = 500 data. It is nevertheless possible to obtain an indication of the Schmidt number dependence by reference to information in the literature on variation of the aeration rate with temperature. Streeter *et al.*[7] give a temperature function

$$k_2'/k_2 = 1{\cdot}047^{T'}/1{\cdot}047^{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

for the temperature variation of k_2 (T in °C).
O'Connor & Dobbins[6] give

$$k_2'/k_2 = 1{\cdot}016^{T'}/1{\cdot}016^{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

while Churchill *et al.*[10] give

$$k_2'/k_2 = 1{\cdot}0241^{T'}/1{\cdot}0241^{T} \quad \ldots\ldots\ldots\ldots\ldots(15)$$

Thus, values for aeration rate vary by about 1—1/2 to 4—1/2% per °C.

The variation of aeration rate with the variation of β, for the same Reynolds number, is

$$\frac{\beta_1}{\beta_2} = \frac{k_1}{k_2}\cdot\frac{\nu_2}{\nu} = \left(\frac{S_2}{S_1}\right)^m = \left(\frac{\nu_2}{\nu_1}\right)^m\cdot\left(\frac{\delta_1}{\delta_2}\right)^m \quad \ldots\ldots\ldots(16)$$

$$\frac{k_1}{k_2} = \left(\frac{S_2}{S_1}\right)^m\cdot\frac{\nu_1}{\nu_2} = \left(\frac{\nu_1}{\nu_2}\right)^{1-m}\cdot\left(\frac{\delta_1}{\delta_2}\right)^m \quad \ldots\ldots\ldots\ldots(17)$$

However, for the same velocity—*not the same Reynolds number*—the law of variation depends on the law of variation of β (or k_2) with Reynolds number. If

$$\beta = C\,\mathrm{Re}^n / S^{-m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

where n has to be estimated from a figure such as Fig. 1, then

$$\frac{\beta_1}{\beta_2} = \frac{k_1}{k_2}\cdot\left(\frac{\nu_2}{\nu_1}\right)^{\frac{1}{2}} = \left(\frac{S_2}{S_1}\right)^m\cdot\left(\frac{\nu_2}{\nu_1}\right)^n \quad \ldots\ldots\ldots\ldots\ldots(19)$$

$$\frac{k_1}{k_2} = \left(\frac{S_2}{S_1}\right)^m\cdot\left(\frac{\nu_1}{\nu_2}\right)^{\frac{1}{2}-n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

The temperature variation in viscosity and diffusivity for this case of gas in water may be determined approximately as follows from Table X.

The difficulty in verifying the Schmidt number dependence by the temperature dependence, one must note, is that it depends on two parameters m and n, unless comparisons are made at the same Reynolds number. From the data in Table X, one notes a temperature coefficient of about −0·052 per °C for the Schmidt number as derived from the interval 26 to 14°C, while the kinematic viscosity has a temperature coefficient of about −0·024 per °C.

TABLE X

Temperature variation effects on kinematic velocity etc.

Temperature, °C	Kinematic viscosity, cm²/sec	Coefficient of diffusion ft²/h	Coefficient of diffusion Relative value	Schmidt number
	(a)	(b)		(c)
0	0·01792			
2	0·01673			
4	0·01567			
6	0·01473			
8	0·01386			
10	0·01308	61	0·753	860
12	0·01237			730
14	0·01171			660
15		71	0·875	
16	0·01112			590
18	0·01057			530
20	0·01007	81	1·00*	480
22	0·00960			430
24	0·00917			390
25		92	1·134	
26	0·00877			360
28	0·00839			
30	0·00804	106	1·309	

(a) 'Handbook of Chemistry and Physics' (b) From reference[6]
* value in cm²/sec $1{\cdot}80\times10^{-5}$ ('Chemical Engineers' Handbook') (c) From reference[10]

One can now estimate the Schmidt number variation with aeration rate from

$$\frac{1}{\beta}\cdot\frac{d\beta}{dT}=\frac{-m}{S}\cdot\frac{dS}{dT}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

$$\frac{1}{k_2}\cdot\frac{dk_2}{dT}=\frac{-m}{S}\cdot\frac{dS}{dT}+\frac{1}{\nu}\cdot\frac{d\nu}{dT}\;\ldots\ldots\ldots\ldots\ldots(22)$$

for the same Reynolds number,

or

$$\frac{1}{\beta}\cdot\frac{d\beta}{dT}=\frac{-m}{S}\cdot\frac{dS}{dT}-\frac{n}{\nu}\cdot\frac{d\nu}{dT}\;\ldots\ldots\ldots\ldots\ldots(23)$$

and

$$\frac{1}{k_2}\cdot\frac{dk_2}{dT}=\frac{-m}{S}\cdot\frac{dS}{dT}+\frac{(\frac{1}{2}-n)}{\nu}\cdot\frac{d\nu}{dT}\;\ldots\ldots\ldots(24)$$

for the same velocity.

The data in the literature cited above for the same velocity gave temperature coefficients of

$$\frac{1}{k_2}\cdot\frac{dk_2}{dT}=0{\cdot}047,\ 0{\cdot}024\text{ and }0{\cdot}016\;\ldots\ldots\ldots\ldots(25)$$

In Fig. 1, for the Streeter data at low Reynolds number, it appears that β is proportional to $Re^{\frac{1}{4}}$ so that $n\sim\frac{1}{4}$.
Thus,

$$\frac{1}{k_2}\cdot\frac{dk_2}{dT}=+0{\cdot}052m-\frac{0{\cdot}024}{4}=0{\cdot}047,\ 0{\cdot}024,\ 0{\cdot}016\;\ldots\ldots(26)$$

The possible values of *m* are:

$$m=1{\cdot}0,\ 0{\cdot}6,\ 0{\cdot}4$$

Thus the range of the temperature data at constant velocity has not pinned down the Schmidt number dependence to any certainty greater than that *m* lies in a range 0·4—1 (as compared with the *a priori* estimate of $\frac{1}{2}$—$\frac{2}{3}$). To determine the Schmidt number dependence more closely, one may compare data for water vapour diffusing across a gas for which the Schmidt number is close to unity and the data for gas diffusing across a water stream for which the Schmidt number is in the 500 range.

Such data are available in an article by Sherwood & Woertz[13] in which the work of Woertz & Towle are cited (also summarised in Sherwood & Pigford 'Absorption and Extraction', 1952, McGraw-Hill). In these experiments evaporating water vapour was allowed to diffuse *across* a rectangular duct, in which there were turbulent flows of air, carbon dioxide and helium. (The water vapour diffused across the channel and was extracted on the other side in equilibrium.)

Their data are the most direct demonstration of the reality of a uniform 'eddy diffusivity' across a turbulent stream, for as they put it, '. . . except for the lowest Reynolds numbers' the eddy diffusivity 'is almost constant over 80—90% of the cross section' (i.e., as one might expect, there are moderate differences in an 'eddy' boundary layer near the walls at which the flow boundary condition of zero axial velocity holds). Furthermore, for their near-unity Schmidt number, the mass-transfer coefficient and turbulent fluid-density product depends only on Reynolds number. The data have been taken from their figure (Fig. 7 in reference[13]) over the half-channel Reynolds number range 5000—50,000. The pertinent data are reproduced in Table XI which was taken from reference.[1]

Data are average for water vapour diffusing across air, carbon dioxide or helium. The depth was taken as a half width (to correspond to rivers) of 1·05 in. and the viscosity as the average for air, carbon dioxide and helium. These data are also plotted in Fig. 1. It appears reasonably clear that the data furnish a most useful extension from the water data to much lower Schmidt number.

It is now possible to estimate the Schmidt number dependence—but, in this case, more directly, at the *same Reynolds* number (not velocity). Choosing a value in the middle of the common range at Re = 10,000 a comparison of the gas data at S = 0·7 and the mean of the liquid data at S ~ 500 gives a value of $m=\frac{2}{3}$ almost exactly. While it is not certain that there is absolute 'parallelism' between the gas and liquid data, the fact that these data have been compared in the transition region suggests that the Schmidt number exponent will be relatively constant. Thus

$$\beta=f(Re)/S^{\frac{2}{3}}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(27)$$

should be a fairly firm form of correlation law.

The temperature coefficient of aeration rate constant for a given velocity should then be given by Equation (24)

and

$$\frac{1}{k_2}\cdot\frac{dk_2}{dT}=\tfrac{2}{3}\times0{\cdot}052-(\tfrac{1}{2}-n)\cdot0{\cdot}024$$

$$=0{\cdot}034-(\tfrac{1}{2}-n)\cdot0{\cdot}024$$

which should change with Reynolds number. Typical values are tabulated from the approximate slopes (values of *n*) obtained from Fig. 1.

Re	*n*, (approx.)	$\frac{1}{k_2}\cdot\frac{dk_2}{dT}$ per °C
10^1	0	0·022
10^2	0·25	0·028
10^3	0·33	0·030
10^4	0·50	0·034
10^5	0·33	0·030
10^6	0·25	0·028
10^7	0·20	0·027

TABLE XI

Evaporative water transferred across tubulent gas streams
(Sherwood & Woertz[13])

K_LH	β	Re
49	0·463	5000
88	0·623	10,000
151	0·755	20,000
264	0·935	40,000
317	1·01	50,000

It appears, quite compatibly, that the Streeter 0·047 coefficient is of the approximate magnitude, although somewhat overstated, and that the Churchill 0·024 coefficient is very close.

It thus appears that the aeration rate dependence for rivers in the Reynolds number range 10^5—10^7 is

$$\beta = C.\mathrm{Re}^{\frac{1}{4}}/S^{\frac{2}{3}} \qquad \text{(28)}$$

or

$$\beta = C'.\mathrm{Re}^{\frac{1}{3}}/S^{\frac{1}{3}} \qquad \text{(29)}$$

Summary and conclusions

A study of aeration rate data, against the background of theory developed in a previous study,[1] has suggested that stream data could be correlated by a correlation law of the form of Equation (14) above.

Comparing gas and liquid data gave the dependence shown in Equation (27), which compares quite well with recent temperature coefficients of aeration given by Churchill *et al.*[10]

At the higher Reynolds number range of 10^5—10^7, as in streams, the correlation is nearly as shown in Equations (28) and (29).

The more general correlation is given in Fig. 1.

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Symbols used

A' = exposed stream area undergoing aeration
A = cross-sectional area of stream
C = constant
H = (stream) depth
K_L = mass transfer 'film' coefficient ($= K_2H$)
K_2 = aeration rate ($k_2 = 0{\cdot}434\ K_2$)
M = mass transfer
P = perimeter of the stream cross-section wetting the walls

Symbols used (continued)

Q = volume flow
R = hydraulic radius (of stream) ($= A/P$)
Re = Reynolds number ($= RV/\nu$)
S = Schmidt number ($= \nu/\delta$)
T = temperature, °C
V = mean stream velocity
a = Adeney entrance coefficient
b = Adeney exit coefficient ($= K_L$)
m, n = exponents used in correlation factors
k = re-aeration rate
p = partial pressure of gas 'diffusing' into the liquid
w = stream width
β = dimensionless re-aeration rate parameter ($= K_LR/\nu.\mathrm{Re}^{\frac{1}{2}}$)
δ = mechanical diffusivity of oxygen into water/or the molecular species which is diffusing across the forced convective stream
ν = kinematic viscosity of water
ρ = concentration of gas in the stream
ρ_s = saturation concentration

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Discussion of Iberall and Cardon, Aeration Mass-Transfer Related to Reynolds Number, J Applied Chemistry

Order Parameter to characterize a system

The figure in Iberall and Cardon (1966) unifies cross channel flows (across the stream) with axial flows, from the microscopic molecular level to the macroscopic. The right half of the S-shaped curve relates to the transport processes carried by large level eddies, as the so-called eddy viscosity measure. These are whirling creations of patterns in rivers, oceans, and galaxies. It also unifies matter and energy diffusion. This unifies data for macro or large collectives of atomistic processes. On the left is the micro diffusion level of molecular processes, known as shear viscosity transport.

It is possible to represent complex hydrodymanic processes by using an order parameter. It was Landau who first talked about order parameters. One order parameter can characterize the entire collective of all flow processes for any general kind of system. In Iberall and Soodak (Yates et al, 1985), the generalized Reynolds number ratio between the macroscopic flow variable to the microscopic diffusional flow variable was used to create an order parameter for both flow pattern and process form (such as transition from gas to liquid or solid state). Whenever a field is overloaded with a flow stress greater than can be absorbed at the micro level, the Reynolds number will indicate that a new formal transformation process will emerge either as a patterned process (function) or a formed process (form). When you pass a criticality condition, a flow field moves to eddy form. In Iberall and Cardon, the flow is from small molecular level to the larger eddies. In Iberall and Soodak, the criticality condition also separates the movement from liquid or gas to solids. Passing the criticality condition can occur either in terms of form or in terms of function.

Below a Reynolds number of 1 (this is a criticality number), you have molecular diffusity. The vertical variable is the ratio of one type of process to another. As its value increases, it goes to another level. This shows the nature of the instability transition from one level to another. At the same time, the Reynolds number is shown to increase 10 million fold. Note that all of the data set of transport in river flows (and most of the chemical engineering processes of engineering interest) lies in the unstable transition zone.

The criticality point precisely indicates where the process being observed changes from being governed by the atomistic levels to the point where it is governed by the collectives. In the Iberall and Cardon figure, for example, on the left is data like those from blood plasma flow going through a capillary. This type of motion is called creeping or laminar flow or smooth flow. On the right is data from turbulent collectives, which are more vigorous and fluctuating in their action. They move bed load in the rivers. They carve channels, move sand, gravel, and even boulders. That turbulent flow could be, for example, the Mississippi River or the Colorado River and how they carve their channels. Or it could relate to a nova or an atom bomb exploding.

Memory function - Contrast simple flow fields to complex flow fields

Simple flow fields have no memory, because there is no internal store of material on which to inscribe their memory function. They do not possess significant memory fields. In the most primitive complex cases, the field memory lies within internal atomic-molecular states within their coupled atomistic interior.

The alternative to simple systems are processes that do possess a memory. These are called thixotropic materials, and macroscopically include materials such as asphalt, paper, paint on wall, silly putty, inks, or sewage. Paper can be bent and stays bent. Silly putty will retain its form. The process gets delayed in the field. There is an extensive time range. A rubber band, for example, can be stretched slowly and it has a low elastic limit. But stretch it fast and it has high elastic limit. It depends on the rate of the strain. Another example is paint. When you paint your walls, you want it to spread nicely and then remain in a fluid sense for 3 to 5 years and not get brittle. You want it to have some slow flow transformability, which makes it thixotropic.

In the figure in Iberall and Cardon, the left bottom represents the external diffusion, of how one local atomism bumped into the next one. But in a complex flow field, there is another type of diffusion that goes into the interior of the atomism. It deals with what we call the internal factory day of the atomism. Thus, in complex systems, there must be a hierarchy of levels. For example, in a complex system like a living human society, a small fraction of the society rules the society ("Through all times, there are those who rule and those who are ruled", Mosca or Sereno, The Ruling Class). In the individual organism, a brain acts as a command-control over the division of function among organs. In the individual organ, the blood borne (endocrine) and nerve borne (unit electrochemical impulses) signals control the functional units in organs (see Bloch & Iberall, "The Functional Unit in Muscle", 1984). The ionophoric gating proteins in cells impose cataytic command-control on the atomic-molecular street flow outside of cells to organize them -- through additional chemical thermodynamic machinery within the cells to manage the complex factory day of the cell. Etc, down to the leptons and quarks.

Or, a similar nested chain descends from the star (Sun) to its solar system (e.g., planets), to planetary systems, to the surface and interior geophysical-geochemical processes, that make the rivers, etc.

All of these hierarchical processes are run by the bulk or associational viscosity which is to be represented within the Reynold number as order parameter.

The order parameter is a measure of the both the external and the internal factory day process. It is the sum of the diffusivity of the inside and the outside. The time scale inside is added to the outside time scale. The basic ideas developed by Landau and Lipshitz would permit you to develop a free energy thermodynamic function for all the major energetic processes captured within the combined internal and external total factory day process. If the inside one is the long term, as in complex atomistic systems, it becomes the dominant process. Look at the ratio between the internal and external diffusion, and forget about the smaller of the two.

Basically, in flowing materials such as water and blood, you create patterns. In solids, you create form. Complex systems tend to be plastic. Their controlling flow systems, at longer scales, tend to yield very slowly.

Implications of this analysis

Most processes we are interested in are in transisition. For example, chemical engineers are very interested in transport processes. They build factories that yield large amounts of steel or process chain or medicines or other materials. But most things are in transition. The implications of things being in the transition zone are that it makes it a lot stranger. But we argue that even though processes and things are mostly in transition, there is no need for a complex theory. It can be explained very simply. In this paper, we discuss how gas, liquids and solid processes can be tied together.

Finding the order number

In physics, there are three primitive states of matter. They comprise gas, liquid, and solid states. That characterization is found at every level in the space-time hierarchy. Each level is characterized by an atomistic and a collective level. These alternate from top to bottom. From the macroscopic to the macroscopic, one can use an order number to describe each of them. From the collective forms, it is possible to determine the order parameter to relate their micro to the macro. Once you have the order parameter, you can get a unique representation of the process. In the Iberall and Cardon figure, one sees the relation between the dynamicaly simple collective of interacting atomisms which gradually transform to become more complex collective phenomena.

Field processes - background

For the characterization at such dynamics collective phenomena, there are only three field processes that come to thermodynamic equilibrium: diffusion, wave propagation, and convection. One can use these terms to describe the totally of all processes in any collective (chemical change, bulk viscosity). After a limited number of interactions, as low as three or four for gases, there is thermodynamic equilibrium.

There are two kinds of processes. The first is a space-like process del (a distance measurement). Del is some small box measure inside the collective. The second is a time process, tau (time). For example, you may blink at tau time. Del is a measure of a near equilibrium atomistic movement, a so-called mean path measurement. Tau can change on breakup of such movement into a time series. You can have only minimal exchanges in tau time.

Things are either coherent or incoherent, logically. In one tau blink, if two players are exchanged between inside and outside the small box in the collective, that is an incoherent exchange (nothing specialized about this) (local). An incoherent exchange is a diffusive process: an exchange at random without specifying a direction. All these exchanges are a prototype. If they change color (or temperature, or spin, or change from hydrogen to oxygen, etc), it is still a diffusive process

On the other hand, if the motion of displacement across the box is in-a-line, this is a coherent process. One atomism enters the box, pushes an atomism which pushes another, and so on, until one leaves the box. For example, a wave propagation in any positive direction (local), is a coherent process. These two are local processes. There is one additional global process.

Another alternative is that you can replace all of the atomisims in the box in a global process. Convection is the model of this. It is a quadratic process, it is the magnitude of the velocity of the field which carries one other box measures. Convection thus will be inherently non-linear.

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